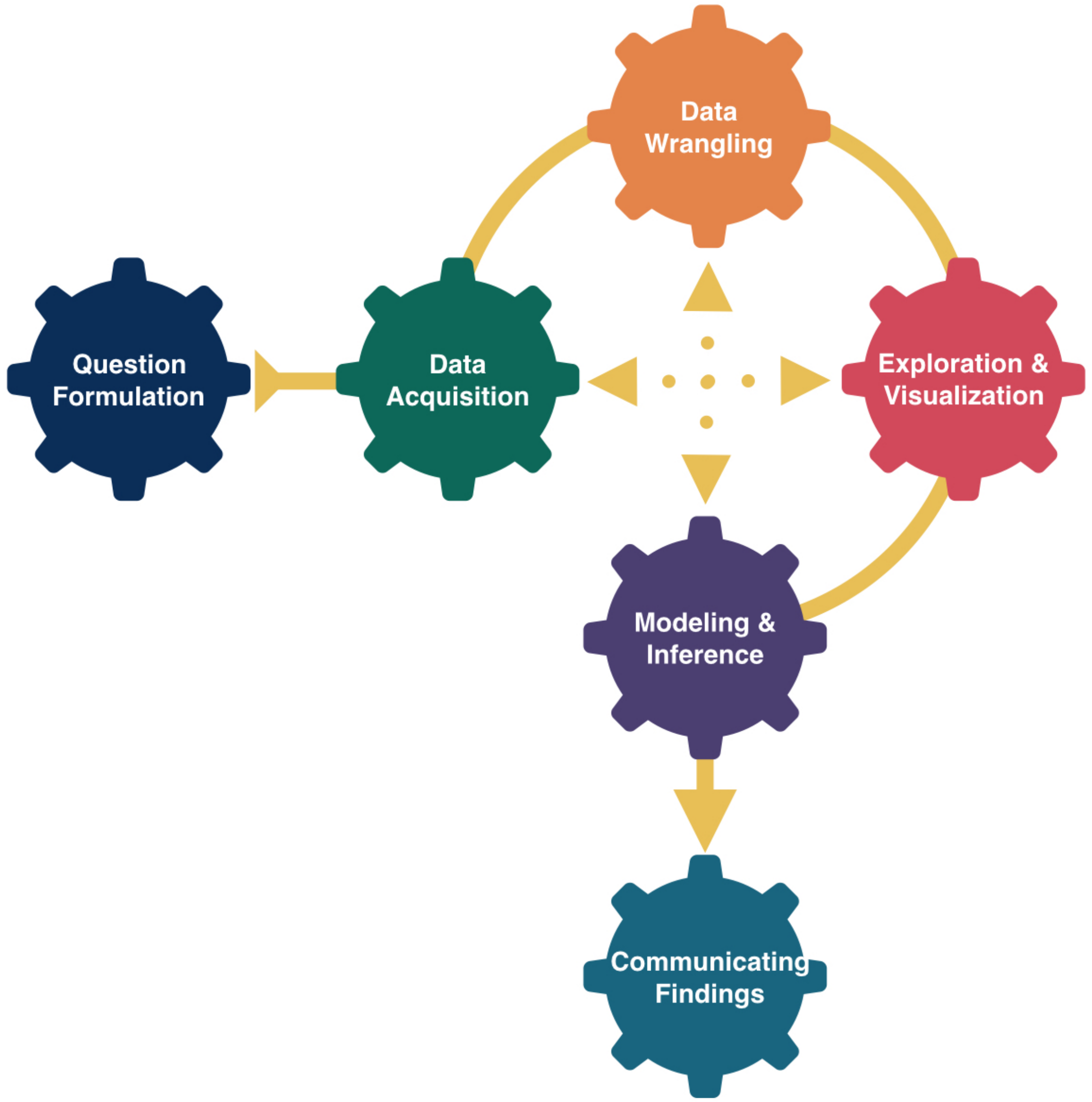


Chi-Squared Tests

Kelly McConville

Stat 100

Week 14 | Fall 2023

Announcements

- Make sure to sign up for an oral exam
- Due Tues, Dec 5th at 5pm:
  - Extra Credit Lecture Quiz
  - P-Set 9
- RSVP for the `ggparty` by end of today: bit.ly/ggpartyf23

Goals for Today

- Comparing models
- What didn't we cover?
- Chi-squared tests
- Wrap-up

Multiple Linear Regression

Linear regression is a flexible class of models that allow for:

- Both quantitative and categorical explanatory variables.
- Multiple explanatory variables.
- Curved relationships between the response variable and the explanatory variable.
- BUT the response variable is quantitative.

How do I pick the **BEST** model?

Comparing Models

Suppose I built 4 different models to predict the price of a Saratoga Springs house. **Which is best?**

```r
library(mosaicData)
mod1 <- lm(price ~ centralAir, data = SaratogaHouses)
mod2 <- lm(price ~ centralAir + waterfront, data = SaratogaHouses)
mod3 <- lm(price ~ centralAir + waterfront + age + livingArea + bathrooms, data = SaratogaHouses)
mod4 <- lm(price ~ centralAir + waterfront + age + livingArea + bathrooms + sewer, data = SaratogaHouses)
```

- Big question! Take **Stat 139: Linear Models** to learn systematic model selection techniques.
- We will explore one approach. (But there are many possible approaches!)

Comparing Models

Suppose I built 4 different model. **Which is best?**

- Pick the best model based on some measure of quality.

Measure of quality: R^2 (Coefficient of Determination)

$$\begin{aligned} R^2 &= \text{Percent of total variation in y explained by the model} \\ &= 1 - \frac{\sum (y - \hat{y})^2}{\sum (y - \bar{y})^2} \end{aligned}$$

Strategy: Compute the R^2 value for each model and pick the one with the highest R^2.

Comparing Models with R^2

Strategy: Compute the R^2 value for each model and pick the one with the highest R^2.

```r
library(mosaicData)
mod1 <- lm(price ~ centralAir, data = SaratogaHouses)
mod2 <- lm(price ~ centralAir + waterfront, data = SaratogaHouses)
mod3 <- lm(price ~ centralAir + waterfront + age + livingArea + bathrooms, data = SaratogaHouses)
mod4 <- lm(price ~ centralAir + waterfront + age + livingArea + bathrooms + sewer, data = SaratogaHouses)
```

Strategy: Compute the R^2 value for each model and pick the one with the highest R^2.

```
library(broom)
glance(mod1)
```

```
# A tibble: 1 × 12
  r.squared adj.r.squared  sigma statistic  p.value    df  logLik    AIC    BIC
      <dbl>         <dbl>  <dbl>     <dbl>    <dbl> <dbl>   <dbl>  <dbl>  <dbl>
1     0.111         0.110 92866.      215. 6.83e-46     1 -22217. 44441. 44457.
# ℹ 3 more variables: deviance <dbl>, df.residual <int>, nobs <int>
```

```
glance(mod2)
```

```
# A tibble: 1 × 12
  r.squared adj.r.squared  sigma statistic  p.value    df  logLik    AIC    BIC
      <dbl>         <dbl>  <dbl>     <dbl>    <dbl> <dbl>   <dbl>  <dbl>  <dbl>
1     0.136         0.135 91536.      136. 1.20e-55     2 -22192. 44392. 44414.
# ℹ 3 more variables: deviance <dbl>, df.residual <int>, nobs <int>
```

```
glance(mod3)
```

```
# A tibble: 1 × 12
  r.squared adj.r.squared  sigma statistic   p.value    df  logLik    AIC    BIC
      <dbl>         <dbl>  <dbl>     <dbl>     <dbl> <dbl>   <dbl>  <dbl>  <dbl>
1     0.560         0.559 65402.      438. 1.02e-303     5 -21610. 43233. 43271.
# ℹ 3 more variables: deviance <dbl>, df.residual <int>, nobs <int>
```

```
glance(mod4)
```

```
# A tibble: 1 × 12
  r.squared adj.r.squared  sigma statistic   p.value    df  logLik    AIC    BIC
      <dbl>         <dbl>  <dbl>     <dbl>     <dbl> <dbl>   <dbl>  <dbl>  <dbl>
1     0.560         0.558 65419.      313. 2.62e-301     7 -21609. 43236. 43285.
# ℹ 3 more variables: deviance <dbl>, df.residual <int>, nobs <int>
```

Problem: As we add predictors, the R^2 value will only increase.

Comparing Models with R^2

Problem: As we add predictors, the R^2 value will only increase.

And in Week 6, we said:

Guiding Principle: Occam's Razor for Modeling

> "All other things being equal, simpler models are to be preferred over complex ones." – ModernDive

Comparing Models with the Adjusted R^2

New Measure of quality: Adjusted R^2 (Coefficient of Determination)

$$\text{adj}R^2 = 1 - \frac{\sum(y-\hat{y})^2}{\sum(y-\bar{y})^2}\left(\frac{n-1}{n-p-1}\right)$$

where p is the number of explanatory variables in the model.

- Now we will penalize larger models.
- **Strategy**: Compute the adjusted R^2 value for each model and pick the one with the highest adjusted R^2.

Strategy: Compute the adjusted R^2 value for each model and pick the one with the highest adjusted R^2.

```r
glance(mod1)
```

```
# A tibble: 1 × 12
  r.squared adj.r.squared  sigma statistic  p.value    df  logLik    AIC    BIC
      <dbl>         <dbl>  <dbl>     <dbl>    <dbl> <dbl>   <dbl>  <dbl>  <dbl>
1     0.111         0.110 92866.      215. 6.83e-46     1 -22217. 44441. 44457.
# ℹ 3 more variables: deviance <dbl>, df.residual <int>, nobs <int>
```

```r
glance(mod2)
```

```
# A tibble: 1 × 12
  r.squared adj.r.squared  sigma statistic  p.value    df  logLik    AIC    BIC
      <dbl>         <dbl>  <dbl>     <dbl>    <dbl> <dbl>   <dbl>  <dbl>  <dbl>
1     0.136         0.135 91536.      136. 1.20e-55     2 -22192. 44392. 44414.
# ℹ 3 more variables: deviance <dbl>, df.residual <int>, nobs <int>
```

```r
glance(mod3)
```

```
# A tibble: 1 × 12
  r.squared adj.r.squared  sigma statistic   p.value    df  logLik    AIC    BIC
      <dbl>         <dbl>  <dbl>     <dbl>     <dbl> <dbl>   <dbl>  <dbl>  <dbl>
1     0.560         0.559 65402.      438. 1.02e-303     5 -21610. 43233. 43271.
# ℹ 3 more variables: deviance <dbl>, df.residual <int>, nobs <int>
```

```r
glance(mod4)
```

```
# A tibble: 1 × 12
  r.squared adj.r.squared  sigma statistic   p.value    df  logLik    AIC    BIC
      <dbl>         <dbl>  <dbl>     <dbl>     <dbl> <dbl>   <dbl>  <dbl>  <dbl>
1     0.560         0.558 65419.      313. 2.62e-301     7 -21609. 43236. 43285.
# ℹ 3 more variables: deviance <dbl>, df.residual <int>, nobs <int>
```

What data structures have we not tackled in Stat 100?

What Else?

Which data structures/variable types are we missing in this table?

Response	Explanatory	Numerical_Quantity	Parameter	Statistic
quantitative	-	mean	μ	$\bar{x}$
categorical	-	proportion	p	$\hat{p}$
quantitative	categorical	difference in means	$\mu_1 - \mu_2$	$\bar{x}_1 - \bar{x}_2$
categorical	categorical	difference in proportions	$p_1 - p_2$	$\hat{p}_1 - \hat{p}_2$
quantitative	quantitative	correlation	ρ	r
quantitative	mix	model coefficients	β_is	$\hat{\beta}_i$s

Inference for Categorical Variables

Consider the situation where:

- Response variable: categorical
- Explanatory variable: categorical
- Parameter of interest: $p_1 - p_2$
  - This parameter of interest only makes sense if **both** variables only have two categories.

It is time to learn how to study the relationship between two categorical variables when **at least one has more than two categories.**

Hypotheses

H_o: The two variables are independent.

H_a: The two variables are dependent.

Example

Near-sightedness typically develops during the childhood years. Quinn, Shin, Maguire, and Stone (1999) explored whether there is a relationship between the type of light children were exposed to and their eye health based on questionnaires filled out by the children's parents at a university pediatric ophthalmology clinic.

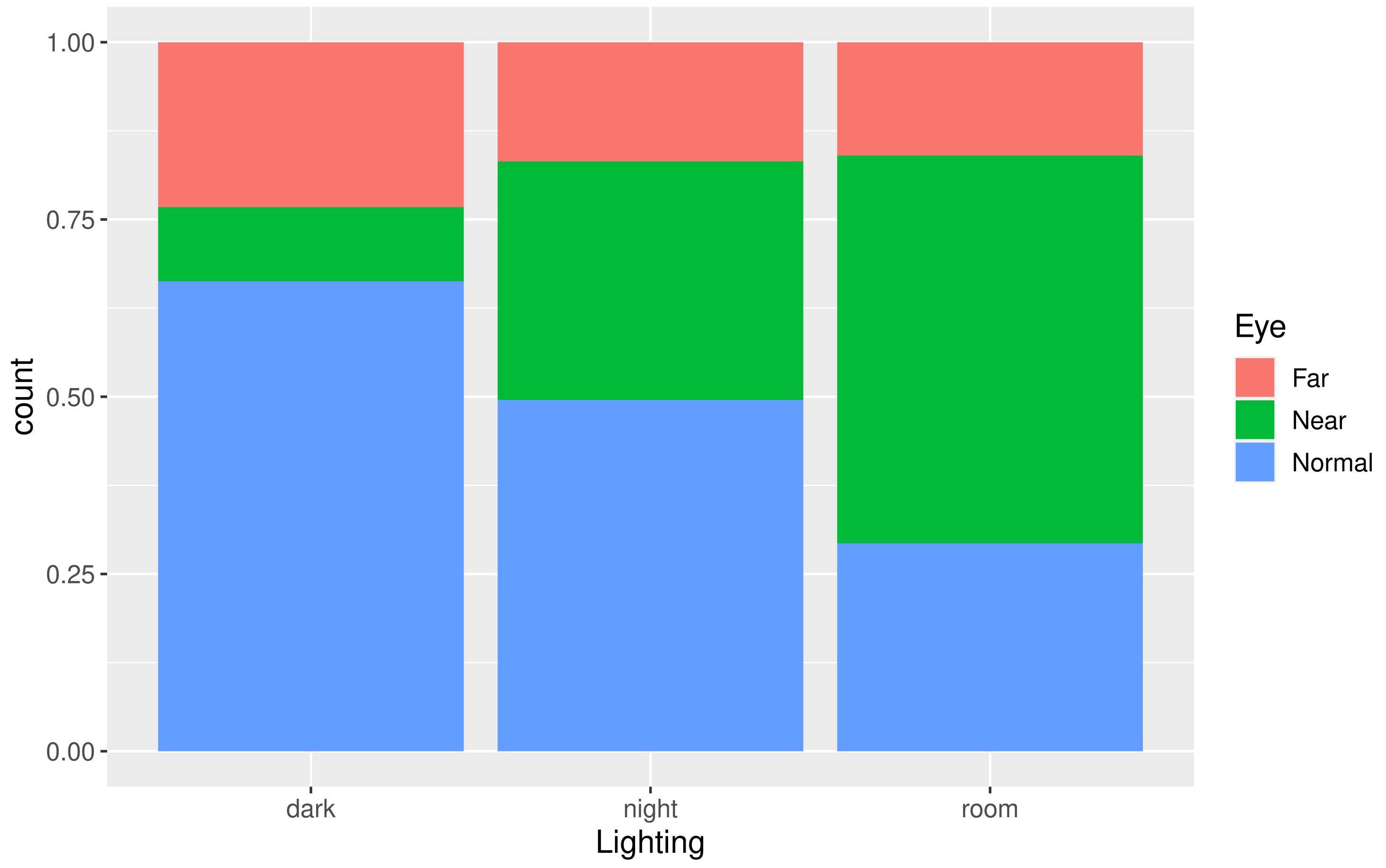
```r
library(tidyverse)
library(infer)

# Import data
eye_data <- read_csv("data/eye_lighting.csv")

# Contingency table
eye_data %>%
  count(Lighting, Eye)
```

```
# A tibble: 9 × 3
  Lighting Eye        n
  <chr>    <chr>  <int>
1 dark     Far       40
2 dark     Near      18
3 dark     Normal   114
4 night    Far       39
5 night    Near      78
6 night    Normal   115
7 room     Far       12
8 room     Near      41
9 room     Normal    22
```

Eyesight Example

Does there appear to be a relationship/dependence?

```
ggplot(data = eye_data,
       mapping = aes(x = Lighting,
                     fill = Eye)) +
  geom_bar(position = "fill")
```



Test Statistic

Need a test statistic!

- Won't be a single sample statistic.
- Needs to measure the discrepancy between the observed sample and the sample we'd expect to see if H_o (no relationship) were true.
- Would be nice if its null distribution could be approximated by a known probability model.

Sample Result Tables

Observed Sample Table

```r
table(eye_data$Eye, eye_data$Lighting) %>%
  addmargins() %>%
  kable(format = "html")
```

	dark	night	room	Sum
Far	40	39	12	91
Near	18	78	41	137
Normal	114	115	22	251
Sum	172	232	75	479

Expected Sample Table

Question: If H_o were correct, is this the table that we'd expect to see?

	dark	night	room	Sum
Far	53	53	53	159
Near	53	53	53	159
Normal	53	53	53	159
Sum	159	159	159	477

Sample Result Tables

Observed Sample Table

```r
table(eye_data$Eye, eye_data$Lighting) %>%
  addmargins() %>%
  kable(format = "html")
```

	dark	night	room	Sum
Far	40	39	12	91
Near	18	78	41	137
Normal	114	115	22	251
Sum	172	232	75	479

Expected Sample Table

Question: If H_o were correct, what table would we expect to see?

Want a H_o table that respects the overall eye condition proportions:

$$\hat{p}_{far} = 91/479$$

$$\hat{p}_{nor} = 251/479$$

$$\hat{p}_{nea} = 137/479$$

Sample Result Tables

Observed Sample Table

```r
table(eye_data$Eye, eye_data$Lighting) %>%
  addmargins() %>%
  kable(format = "html")
```

	dark	night	room	Sum
Far	40	39	12	91
Near	18	78	41	137
Normal	114	115	22	251
Sum	172	232	75	479

Expected Sample Table

Question: If H_o were correct, what table would we expect to see?

	dark	night	room	Sum
Far	(91/479)172	(91/479)232	(91/479)75	91
Near	(137/479)172	(137/479)232	(137/479)75	137
Normal	(251/479)172	(251/479)232	(251/479)75	251
Sum	172	232	75	479

Still have the same totals but distributed the values differently within the table

Sample Result Tables

Observed Sample Table

```r
table(eye_data$Eye, eye_data$Lighting) %>%
  addmargins() %>%
  kable(format = "html")
```

	dark	night	room	Sum
Far	40	39	12	91
Near	18	78	41	137
Normal	114	115	22	251
Sum	172	232	75	479

Expected Sample Table

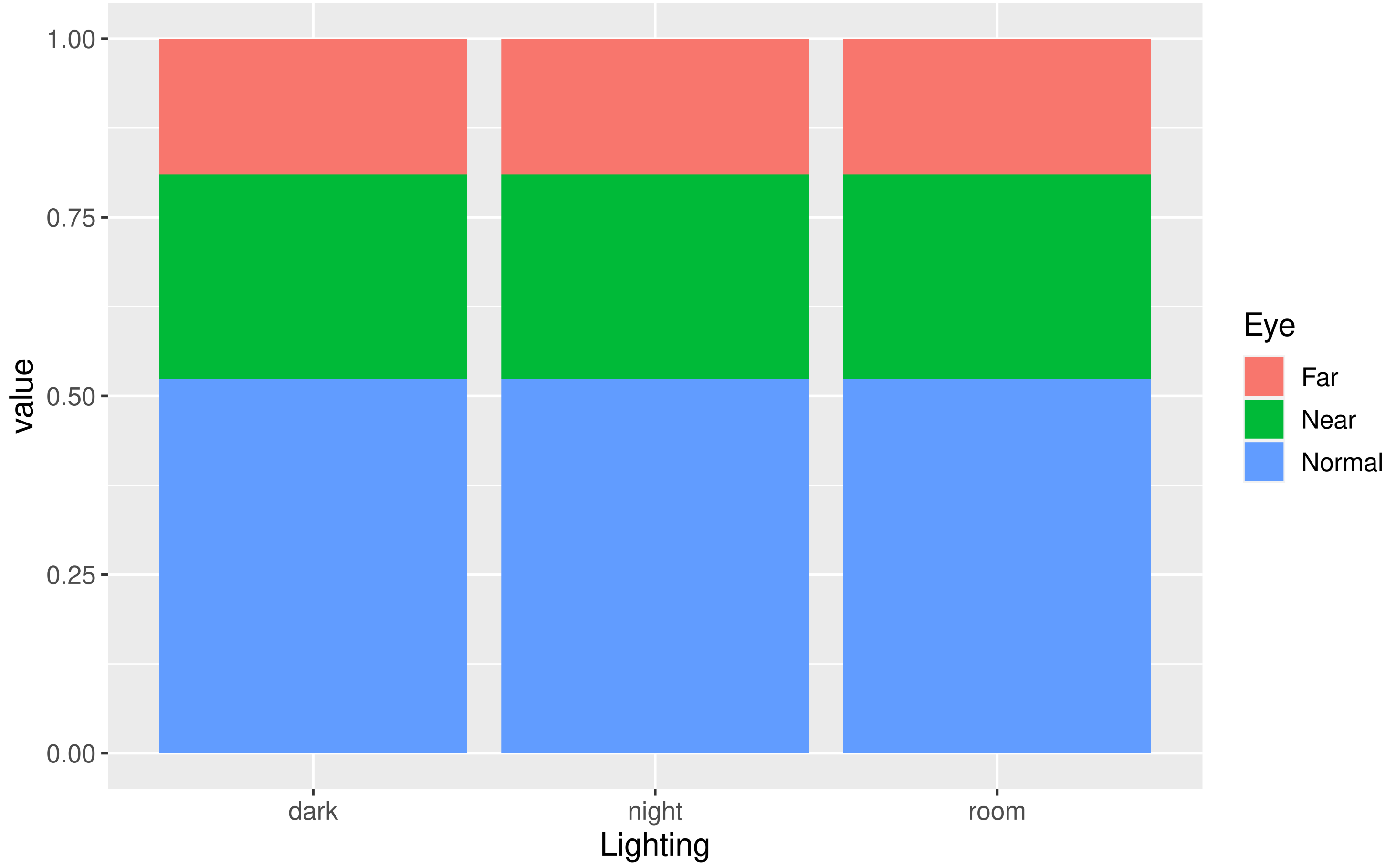
Question: If H_o were correct, what table would we expect to see?

	dark	night	room	Sum
Far	32.7	44.1	14.2	91.0
Near	49.2	66.4	21.5	137.1
Normal	90.1	121.6	39.3	251.0
Sum	172.0	232.1	75.0	479.1

Expected Table

How does this table represent H_o?

	dark	night	room	Sum
Far	32.68	44.08	14.25	91.01
Near	49.19	66.35	21.45	136.99
Normal	90.13	121.57	39.30	251.00
Sum	172.00	232.00	75.00	479.00



Test Statistic

Want the test statistic to quantify the difference between the observed table and the expected table.

	dark	night	room	Sum
Far	40	39	12	91
Near	18	78	41	137
Normal	114	115	22	251
Sum	172	232	75	479

	dark	night	room	Sum
Far	32.68	44.08	14.25	91
Near	49.19	66.35	21.45	137
Normal	90.13	121.57	39.30	251
Sum	172.00	232.00	75.00	479

For each cell: Compute a **Z-score**!

$$\begin{aligned}\text{Z-score} &= \frac{\text{stat - mean}}{\text{SE}} \\ &= \frac{\text{observed - expected}}{\sqrt{\text{expected}}}\end{aligned}$$

Test Statistic

Want the test statistic to quantify the difference between the observed table and the expected table.

	dark	night	room
Far	1.28	-0.76	-0.60
Near	-4.45	1.43	4.22
Normal	2.51	-0.60	-2.76

Test Statistic

Test Statistic Formula:

$$\chi^2 = \sum \left(\frac{\text{observed - expected}}{\sqrt{\text{expected}}} \right)^2$$

```r
library(infer)
#Compute Chi-square test stat
test_stat <- eye_data %>%
  specify(Eye ~ Lighting) %>%
  calculate(stat = "Chisq")
test_stat
```

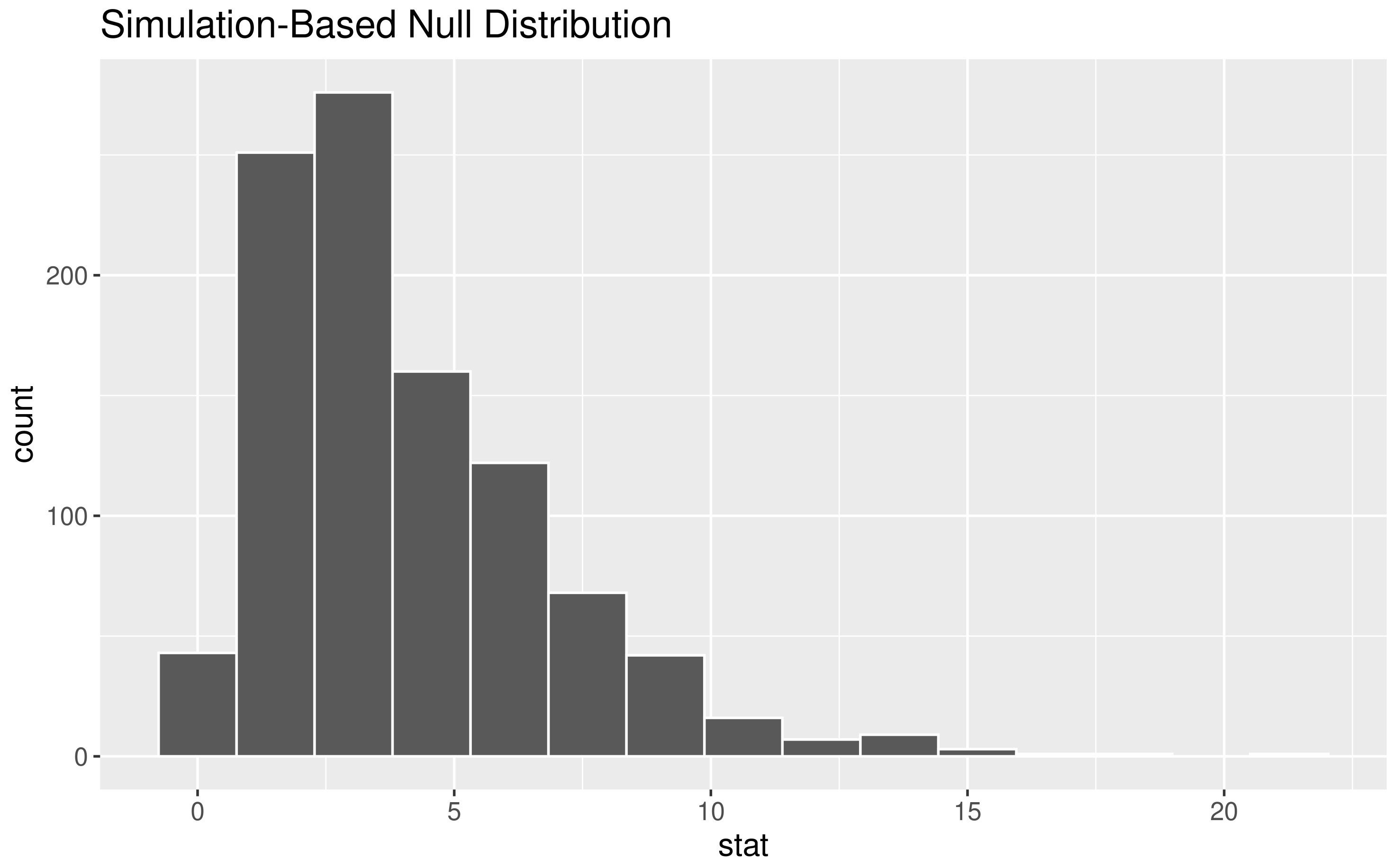
```
Response: Eye (factor)
Explanatory: Lighting (factor)
# A tibble: 1 × 1
   stat
  <dbl>
1  56.5
```

Questions:

1. Is a test statistic unusual if it is a large number or a small number?
2. Is 56.5 unusual under H_o?

Generating the Null Distribution

```
# Construct null distribution
null_dist <- eye_data %>%
  specify(Eye ~ Lighting) %>%
  hypothesize(null = "independence") %>%
  generate(reps = 1000, type = "permute") %>%
  calculate(stat = "Chisq")


visualize(null_dist)
```



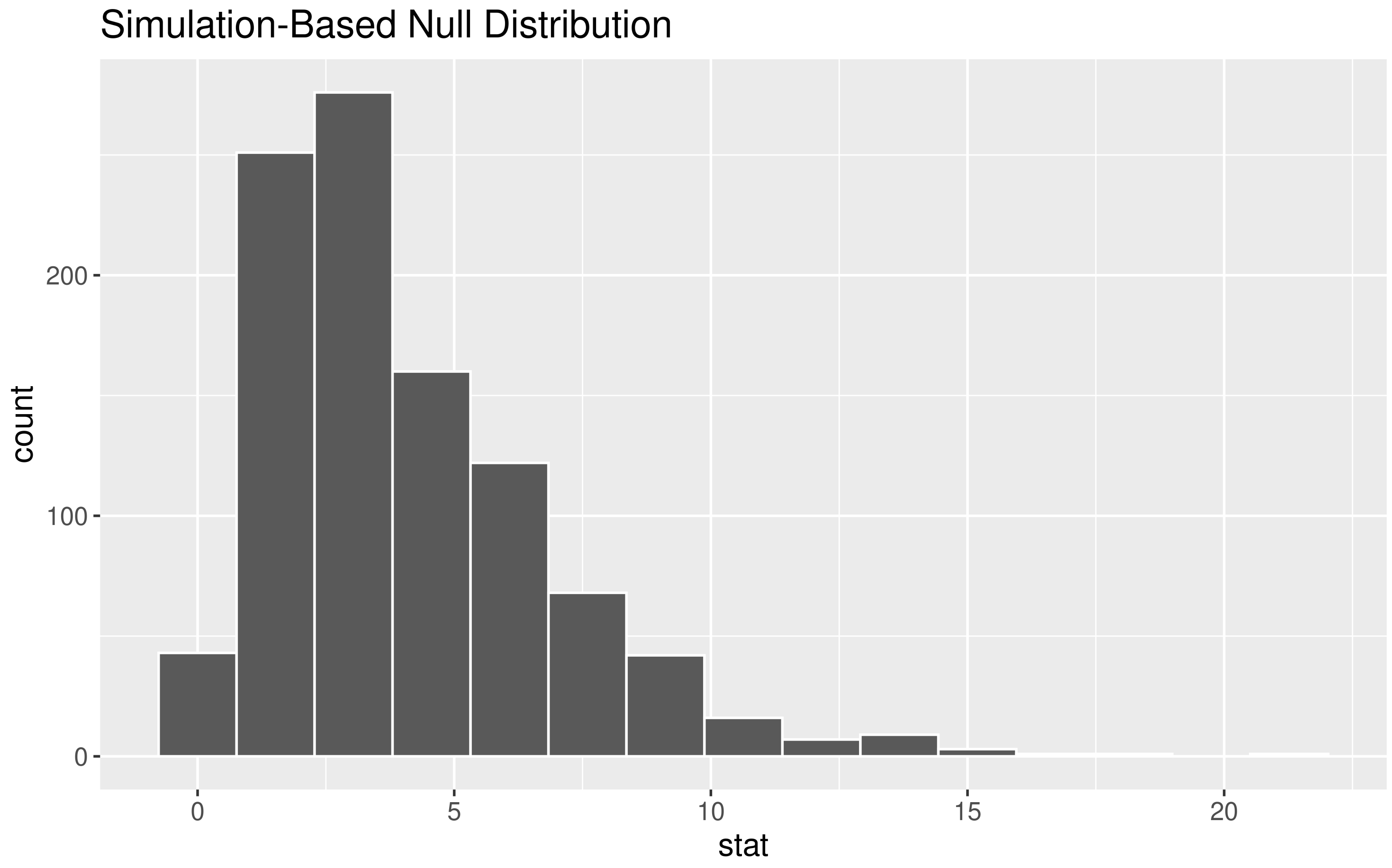
The Null Distribution

Key Observations about the distribution:

Smallest possible value?

Shape?

Is our observed test statistic of 56.5 unusual?



The P-value

```r
# Compute p-value
null_dist %>%
  get_pvalue(obs_stat = test_stat, direction = "greater")
```

```
# A tibble: 1 × 1
  p_value
    <dbl>
1       0
```

Approximating the Null Distribution

If there are at least 5 observations in each cell, then

$$\text{test statistic} \sim \chi^2(df = (k-1)(j-1))$$

where k is the number of categories in the response variable and j is the number of categories in the explanatory variable.

The df controls the center and spread of the distribution.

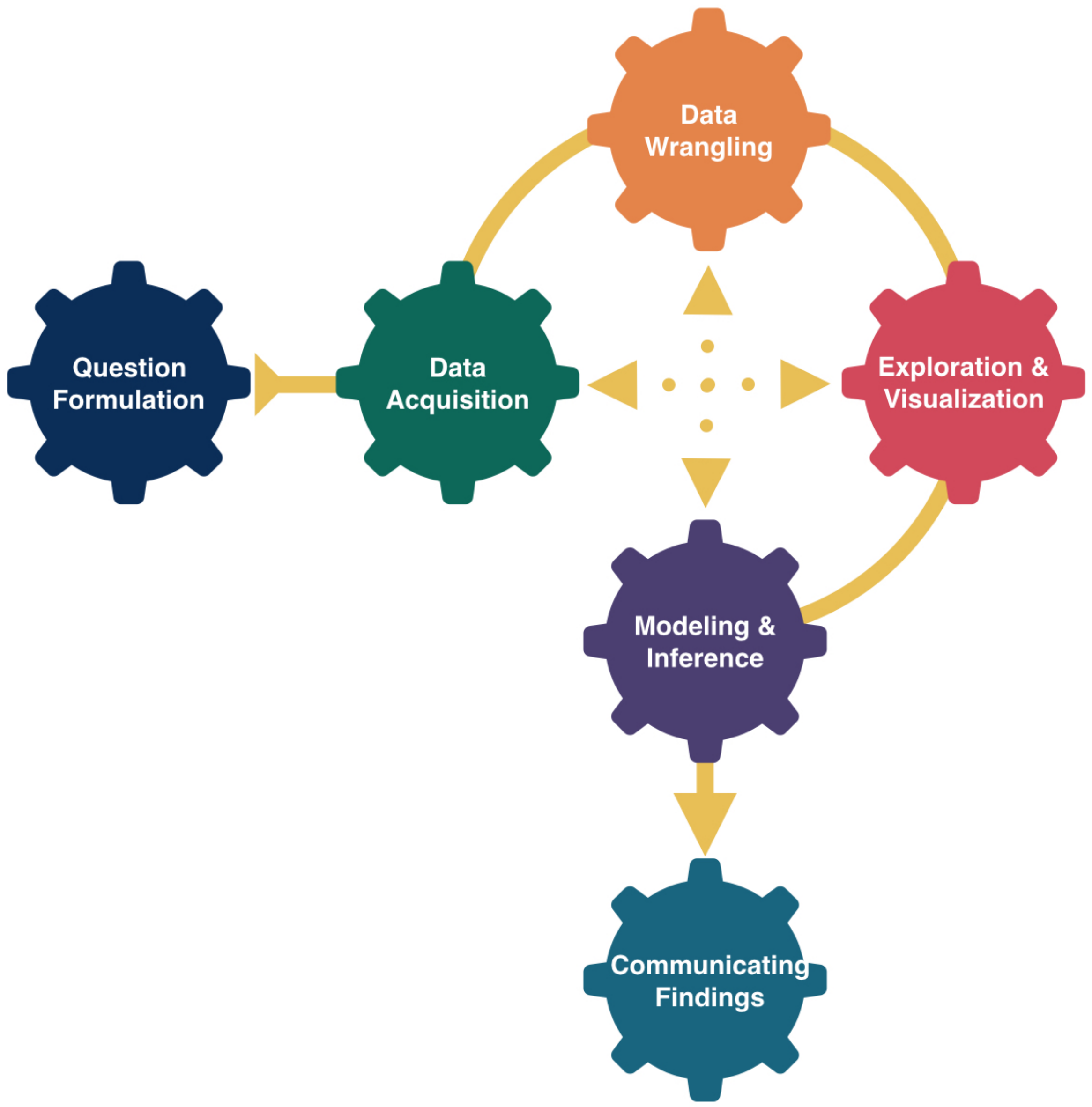
The Chi-Squared Test

```r
chisq.test(table(eye_data$Eye, eye_data$Lighting))
```

```
	Pearson's Chi-squared test

data:  table(eye_data$Eye, eye_data$Lighting)
X-squared = 56.513, df = 4, p-value = 1.565e-11
```

- Conclusions?
- Causal link between room lighting at bedtime and eye conditions?
- Decisions, decisions…

(Some of the) Course Learning Objectives



- Learn how to **analyze** data in R.
- Master **creating** graphs with `ggplot2`.
- Apply data wrangling operations with `dplyr`.
- Translate a research problem into a set of relevant questions that can be answered with data.
- Reflect on how **sample design structures** impact potential conclusions.
- Appropriately apply and draw inferences from a statistical model, including **quantifying and interpreting the uncertainty** in model estimates.
- Develop a **reproducible** workflow using R Quarto documents.

Checklist of Remaining Stat 100 Items

🦜 Sign up for an oral exam slot.

✅ Finish P-Set 9 by 5pm on Tuesday.

⭐ Complete the Extra Credit Lecture Quiz by 5pm on Tuesday.

❓ Come by office hours with any questions while studying for the final exam.

‼️ Complete the oral exam on Dec 13th or 14th and the in-class on Dec 15th at 9am - noon.

🎉 Attend the `ggparty` on Thursday at noon in SC 316.

📊 Consider what other stats classes to take now that I am 25% of the way to a stats secondary.

🌲 Add a calendar note to email Prof McConville on 12/4/33 to show her I still know how to interpret a p-value.

Thanks for a wonderful semester!